Shadow pole contribution to the S-matrix in potential scattering

1974 J. Phys. A: Math. Nucl. Gen. 7 2267

(http://iopscience.iop.org/0301-0015/7/18/006)

J. Phys. A: Math., Nucl. Gen., Vol. 7, No. 18, 1974. Printed in Great Britain. © 1974

Shadow pole contribution to the S-matrix in potential scattering

G Sethia, M C Gupta and C S Shastry
Department of Physics, Birla Institute of Technology and Science, Pilani (Rajasthan), India

Received 5 August 1974

Abstract. The contributions of 'shadow poles' to the nonrelativistic S-wave scattering by the exponential and the Hulthen potentials are calculated and the implications of these results in the theory based on 'shadow poles' in particle physics are discussed.

1. Introduction

In the analytic S-matrix theory, one seeks to interpret the scattering phenomena in terms of the singularities of the S-matrix in the complex momentum, energy or angular momentum planes. In general, in the principal sheet of the complex energy plane, the S-matrix contains all those singularities corresponding to which the Schrödinger equation has solutions which belong to the L^2 class and these form a complete set. These singularities are the same as those occurring in the analytic structure of the Green function in the principal sheet of the complex energy plane. In other words, the Green function and the S-matrix have singularities which correspond to the spectrum of the Hamiltonian. However, the S-matrix can have additional singularities in the principal sheet of the complex energy plane which have dynamical characteristics in the sense that they do contribute to the scattering cross section but the solution of the Schrödinger equation corresponding to these does not belong to the spectrum of the Hamiltonian. Sudarshan and co-workers (1970) have quite recently developed the concept of 'shadow states' corresponding to these singularities and initiated the field theoretical models for elementary particle interactions by taking into account the shadow states. The investigation of the non-L^2 nature of the solutions corresponding to shadow poles is carried out by Nelson *et al* (1971). Nelson and Rajagopal and Sudarshan and Nelson (1972) have done further work on the quantum field theory of 'shadow poles'.

Just as in the case of Regge pole theory, the results of potential scattering theory are the guiding principles in developing the field theoretical models based on shadow poles. It is therefore important to examine the importance of shadow poles and their contribution to scattering cross section in potential scattering. With this in view, we report here the results of calculations which indicate the relative importance of shadow poles in potential scattering theory. In nonrelativistic S-wave scattering, the S-matrices corresponding to the exponential and the Hulthen potentials have shadow poles in addition to bound state poles in the complex energy plane. We calculate the S-matrix for these two potentials and we also calculate the contribution of the shadow poles to the S-matrix for various strengths and ranges of potentials. These results will indicate the relative significance of the contribution of the non-L^2 class poles (shadow poles) and the singularities corresponding to the spectrum of the Hamiltonian in various regions of energy and for different strengths and ranges of potentials. In § 2 we report the results of our calculations and summarize the conclusions.

2. Shadow pole contribution in S-wave scattering by the exponential and the Hulthen potentials

Let us first consider the exponential potential

$$V(r) = -V_0\, \mathrm{e}^{-r/a}.$$

In this case, the S-wave S-matrix is

$$S_0(k) = \frac{\Gamma(1+\nu)}{\Gamma(1-\nu)}\left(\frac{2}{z}\right)^{2\nu} \frac{J_\nu(z)}{J_{-\nu}(z)}$$

where $z = 2au_0^{1/2}$, $u_0 = 2mV_0/\hbar^2$ and $\nu = 2\mathrm{i}ak$. The Jost function (Newton 1966) corresponding to the S-matrix is

$$f(k) = \exp[\mathrm{i}ak\ln(a^2u_0)]\Gamma(1-2\mathrm{i}ak)J_{-2\mathrm{i}ak}(2au_0^{1/2}).$$

The term $\Gamma(1-2\mathrm{i}ak)$ contains all the shadow poles that occur at the points $2\mathrm{i}ka = -1, -2, \ldots$. Then we write the S-matrix as the product

$$S = S_\mathrm{p}\bar{S}$$

where

$$S_\mathrm{p} = \frac{\Gamma(1+2\mathrm{i}ak)}{\Gamma(1-2\mathrm{i}ak)}$$

and

$$\bar{S} = \left(\frac{2}{z}\right)^{2\nu} \frac{J_\nu(z)}{J_{-\nu}(z)}.$$

Here S_p is the contribution of the shadow poles in the product representation for the S-matrix. $\bar{S}$ is the remaining contribution.

Similarly in the case of the Hulthen potential

$$V(r) = \frac{V_0\, \mathrm{e}^{-r/a}}{1-\mathrm{e}^{-r/a}}$$

the S-matrix is

$$S = S_\mathrm{p}\bar{S}$$

where

$$S_\mathrm{p} = \frac{\Gamma(1+2\mathrm{i}ak)}{\Gamma(1-2\mathrm{i}ak)}$$

and

$$\bar{S} = \frac{\Gamma(1+A)\Gamma(1+B)}{\Gamma(1-A)\Gamma(1-B)}$$

where

$$A = -\mathrm{i}ak + \mathrm{i}a(k^2 + u_0)^{1/2}$$

$$B = -\mathrm{i}ak - \mathrm{i}a(k^2 + u_0)^{1/2}.$$

The Jost function (Newton 1966) corresponding to the S-matrix is

$$f(k) = \frac{\Gamma(1-2\mathrm{i}ak)}{\Gamma(1+B)\Gamma(1+A)} = \prod_{n=1}^{\infty}\left(1+\frac{a^2 u_0}{n(n-2\mathrm{i}ak)}\right).$$

We calculate S and S_{p} for various V_0 and 'a' over the range $k = 0{\cdot}2$ to $k = 4{\cdot}0$ in order to estimate the contribution of S_{p} to the S-matrix, and then calculate the S-wave differential cross section.

In figure 1, we plot the exact S-wave differential cross section σ_{S} for the attractive exponential potential, together with the cross section $\bar{\sigma}_{\mathrm{S}}$ corresponding to the phase shift $\bar{\delta}_0 = \delta_0 - \delta_0^{(\mathrm{p})}$ where δ_0 is the exact S-wave phase shift and $\delta_0^{(\mathrm{p})}$ is the phase shift corresponding to S_{p}. These two sets of graphs indicate the difference in the differential

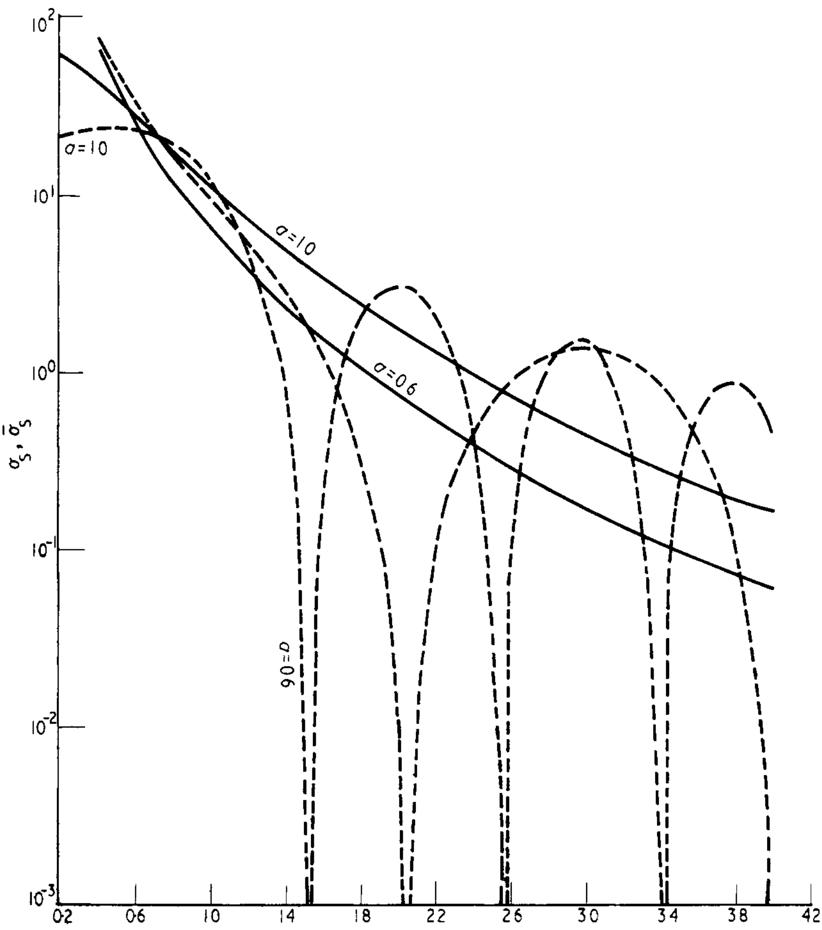

Figure 1. σ_{S} (full curves) and $\bar{\sigma}_{\mathrm{S}}$ (broken curves) for attractive exponential potential for $V_0 = 4{\cdot}0$, $a = 0{\cdot}6$ and for $V_0 = 4{\cdot}0$, $a = 1{\cdot}0$.

cross sections σ_S and $\bar{\sigma}_S$. In figure 2 similar graphs are shown for the repulsive exponential potential. Figure 3 gives the σ_S and $\bar{\sigma}_S$ for the case of the repulsive Hulthen potential.

The differential cross section σ_S decreases monotonically with k, whereas $\bar{\sigma}_S$ has large fluctuations. In fact $\bar{\sigma}_S$ goes to zero at different values of k and these correspond to the case when $\delta_0 = \delta_0^{(p)}$. This indicates that in those problems where the shadow pole contribution is important, one will end up with anomalous behaviour in the differential cross section if the contributions from the shadow poles are not taken into account appropriately.

The role of shadow poles in the exponential or the Hulthen potential is taken by the 'unphysical cut' in the left hand E-plane in the case of superposition of the Yukawa potential. Our above calculations seem to indicate that the roles of either the shadow poles or, correspondingly, the unphysical cut are very important in potential scattering and hence are likely to be equally important in their generalization in particle physics.

It should however be mentioned that in the nonrelativistic scattering problem where the Schrödinger equation is usually numerically solved to obtain the phase shift, the

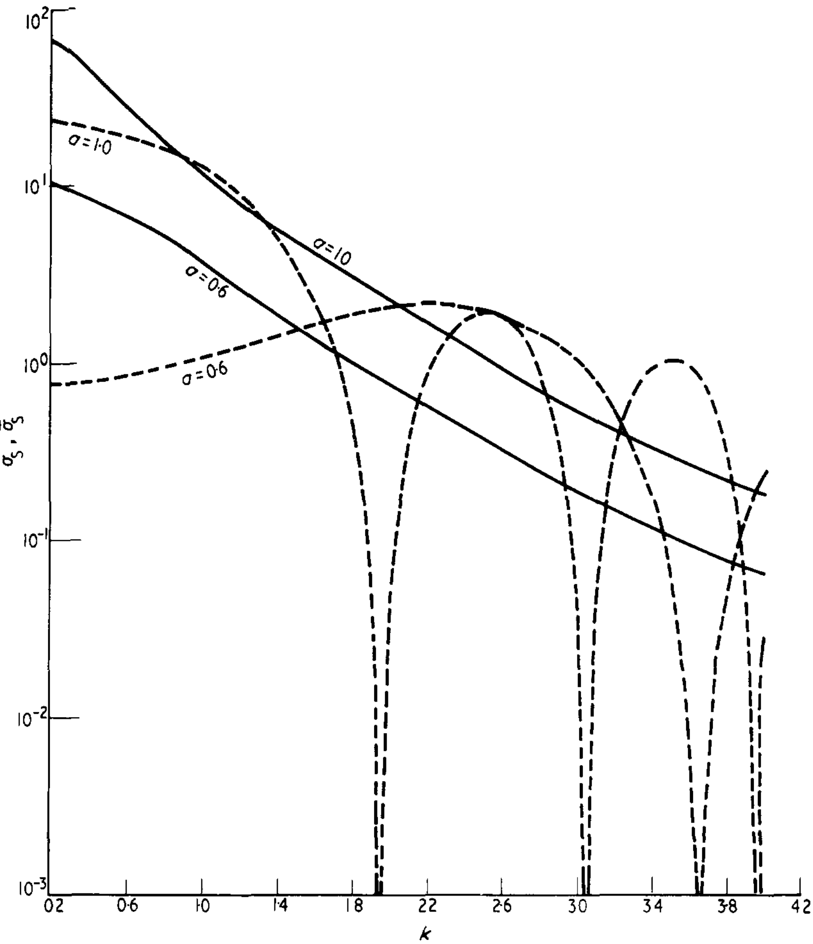

Figure 2. σ_S (full curves) and $\bar{\sigma}_S$ (broken curves) for repulsive exponential potential for $V_0 = 4.0$, $a = 0.6$ and for $V_0 = 4.0$, $a = 1.0$.

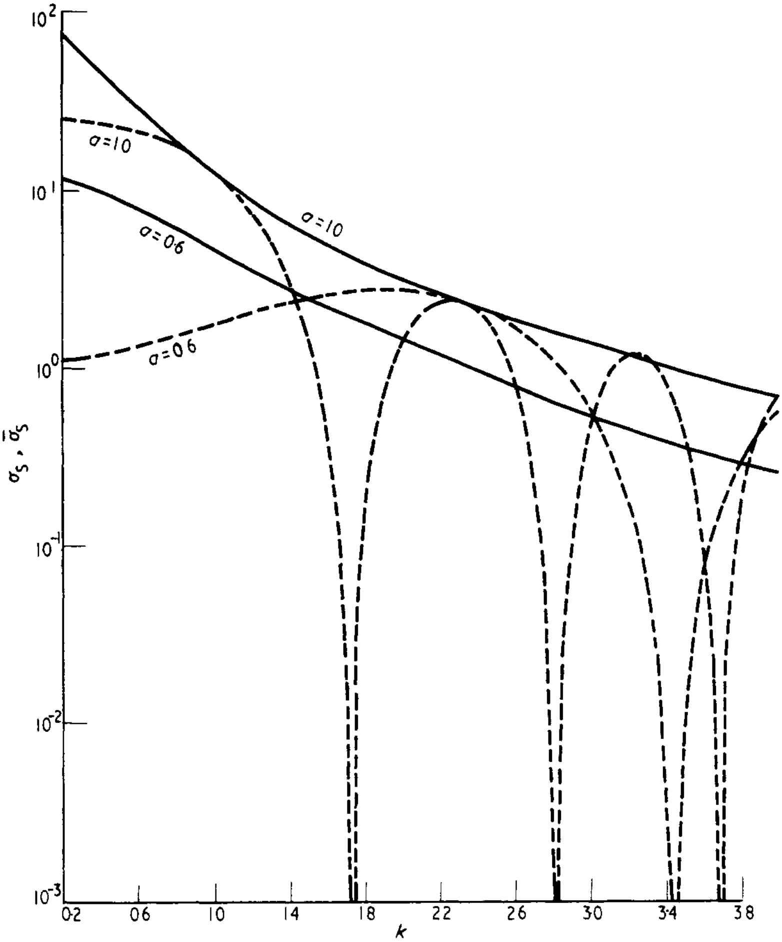

Figure 3. σ_S (full curves) and $\bar{\sigma}_S$ (broken curves) for the repulsive Hulthen potential for $V_0 = 4.0$, $a = 0.6$ and for $V_0 = 4.0$, $a = 1.0$.

necessity of separating the contribution of various categories of poles (and perhaps cuts) does not usually arise, but this is not the case in S-matrix theory and dispersion relations. Our simple calculations indicate that shadow poles, even though they do not correspond to the L^2 class of solutions (Sudarshan *et al* 1970 and Nelson *et al* 1971), have significant contributions to the cross section. Details of the field theories based on the shadow states are described by Nelson *et al* (1971) and Sudarshan and Nelson (1972).

Acknowledgments

The authors wish to express their sincere thanks to Dr S Kumar for useful discussions. One of us (MCG) acknowledges the financial assistance provided by UGC, Government of India. The authors wish to thank the IPC staff of BITS for providing computation facilities.

References

Nelson C A, Rajagopal A K and Shastry C S 1971 *J. Math. Phys.* **12** 737–41
Newton R G 1966 *Scattering Theory of Waves and Particles* (New York: McGraw Hill) pp 420–1
Sudarshan E C G, Biswas S N and Pradhan T 1970 *University of Texas at Austin preprint* (September 1970)
Sudarshan E C G and Nelson C A 1972 *Phys. Rev.* D **6** 3658–78, 3678–88